MATH 121A Prep: Row Operations

Facts to Know:

Elementary Row Operations:

1. Swap rows ex: $R1 \leftrightarrow R2$ "swap row 1 with row 2"
2. Multiply row by nonzero scalar ex: $R1 = cR1$ "multiply row 1 by constant c"
3. Add multiple of one row to another ex: $R2 = R2 + 3R1$ "multiply row 1 by 3 and add the result to row 2"

Solving Systems of Linear Equations: $A\vec{x} = \vec{b}$

(1) Form augmented matrix $[A \mid \vec{b}]$

(2) Put in row echelon form or reduced row echelon form

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & * & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Examples:

1. Put the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 4 & 2 \end{bmatrix}$ in row echelon form.

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 4 & 2 \end{bmatrix} \xrightarrow[R3 = R3 + R1]{R2 = R2 - 2R1} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 2 \\ 0 & 6 & 1 \end{bmatrix}$$

$$\xrightarrow{R2 = -R2} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -2 \\ 0 & 6 & 1 \end{bmatrix} \xrightarrow{R3 = R3 - 6R2} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 13 \end{bmatrix}$$

$$\xrightarrow{R3 = \frac{1}{13}R3} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

row echelon form!

2. Solve the matrix equation $A\vec{x} = \vec{b}$ where $A = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 1 & 3 \\ -2 & 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ -3 \\ -5 \end{bmatrix}$

A $\vec{b}$

$$\left[\begin{array}{ccc|c} 2 & -2 & 1 & 3 \\ 0 & 1 & 3 & -3 \\ -2 & 1 & 1 & -5 \end{array}\right] \xrightarrow{R3 = R3 + R1} \left[\begin{array}{ccc|c} 2 & -2 & 1 & 3 \\ 0 & 1 & 3 & -3 \\ 0 & -1 & 2 & -2 \end{array}\right]$$

$$\xrightarrow{R1 = \frac{1}{2}R1} \left[\begin{array}{ccc|c} 1 & -1 & 1/2 & 3/2 \\ 0 & 1 & 3 & -3 \\ 0 & -1 & 2 & -2 \end{array}\right] \xrightarrow{R3 = R3 + R2} \left[\begin{array}{ccc|c} 1 & -1 & 1/2 & 3/2 \\ 0 & 1 & 3 & -3 \\ 0 & 0 & 5 & -5 \end{array}\right]$$

$$\xrightarrow{R3 = \frac{1}{5}R3} \left[\begin{array}{ccc|c} 1 & -1 & 1/2 & 3/2 \\ 0 & 1 & 3 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$x_1 - x_2 + \frac{1}{2}x_3 = 3/2 \rightarrow x_1 - \frac{1}{2} = \frac{3}{2} \rightarrow x_1 = 2$

$x_2 + 3x_3 = -3 \rightarrow x_2 - 3 = -3 \rightarrow x_2 = 0$

$x_3 = -1$

$$\vec{x} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

3. For what values of a does the matrix equation

$$\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ a \end{bmatrix}$$

have a solution?

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ -2 & -4 & a \end{array}\right] \xrightarrow{R2 = R2 + 2R1} \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & a+6 \end{array}\right]$$

x_1

$0x_1 + 0x_2 = a + 6$

$0 = a + 6 \rightarrow \boxed{a = -6}$